Quiz 2

1) What is aliasing and Nyquist frequency?

Aliasing is a phenomenon that emerges when attempting to measure a signal. Specifically, it describes distortions and loss of signal information due to having an insufficiently high sampling rate. For example, imagine you are watching the propeller on an airplane begin to move and start to speed up. At first you can see it perform a complete rotation. However, at some point the motion of the propellor blade starts blending together and almost starts to look like a disk. Maybe you can still detect some motion, but you can no longer see exactly where the propellor blade is along its path of rotation like you did at lower rotations per some unit time. This is an example of aliasing. The signal that your eyes are processing has exceeded their sampling rate (about 30 Hz), and thus your reconstruction of the signal has begun to produce unreliable information.

Related to aliasing, the Nyquist frequency is the highest frequency that can be reconstructed without distortion or loss of information. It is given to us by the relation -- Nyquist frequency = 0.5 x Sampling rate. What this means is that if our eyes have a natural sampling rate of 30 Hz, then the highest frequency they can accurately process is 15 Hz. To continue with our example, a the propellor blade of a plane typically rotates around 1600 rpm, or roughly 30 Hz. Since the Nyquist frequency of our eyes is 15 Hz, this explains why we are unable to process their full motion at their top speed of rotation.

2) What is Gibbs phenomenon?

When we use Fourier series to approximate a function, we commonly see parts where we over or under shoot the function we are trying to approximate. These jumps become particularly pronounced if a function has discontinuities. While in theory we can always add more terms to reduce our approximation’s inaccuracy, the jumps at these discontinuities never completely go away, instead becoming narrower and narrower spiked equally about 9% the height of the jump. This is called the Gibbs phenomenon.

3) Explain orthogonal functions.

Two functions are said to be orthogonal if the result of their inner product is zero. Specifically, orthogonality tells us that the functions are independent and do not correlate or overlap over some interval. This is useful since we can then use the orthogonal functions to set up a linear combination with which all other functions can be represented over the interval of orthogonality. An example of this is how we use the orthogonal sine and cosine function in a linear combination to approximate other functions like we do in the Fourier series.

4) Mathematically what do you know about sine and cosine function?

Sine and cosine are two orthogonal functions which represent the y and x components of a unit circle respectively. Each oscillates over with some frequency in between the bounds of some amplitude set by a constant or function the is multiplied to them. When evaluated a 0, sine is equal to and starts to increase, while cosine starts at its maximum and starts to decrease. They can each be offset by some phase angle, which can even make them equal to one another; e.g. sin(x + pi/2) = cos(x).

Their status as orthogonal functions make them extremely useful in breaking down vectors in planes, as each component of a vector in two dimensions can be represented by the product of its magnitude and either sine or cosine. They can also be used to represent every other kind of trigonometric function. For example, sine/cosine = tangent; 1/cosine = secant; 1/sine = cosecant; etc….

5) Solve 𝑦′′ + 𝑘𝑛+ 2𝑦 = 0, for 0 ≤ 𝑥 ≤ 𝑙, with boundary conditions of 𝑦(𝑥 = 0) = 𝑦(𝑥 = 𝑙) = 0.

6) What did you learn in this chapter? Explain briefly.

**James:** I thought learning about the Gibbs phenomenon was fascinating, since it was something that I noticed when using Fourier series on my own. While the spikes around the jump at a discontinuity do become narrower and narrower the more terms are added and the error that add becomes increasing negligible, it still seems makes me feel uneasy having a flaw in the math that doesn’t go away, even as we go to infinity. Right now, as I conceptualize it, the Gibb phenomenon is the result of attempting to describe a discontinuity in terms of a continuity. So effectively, the source of the error comes about because we are translating something that isn’t fully translatable, just translatable enough.